A Closed-Form Formulation for the Total Power Radiated by a Single-Wire Overhead Line

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***Abstract*— A new analytical solution for the total power radiated by a single-wire line over a ground-plane is here proposed, for the special case of a lossless complex exponential current-distribution. The line is not assumed to be electrically short or "closed", thus being suitable for assessing the emission/immunity of actual transmission-lines as in the automotive domain. A practical procedure employing this model is proposed and it is shown to excellently agree with the experimental results obtained in a mode-stirred reverberation chamber. An application to unshielded multi-conductor transmission lines (MTL) is also proposed.**

I. Introduction

The transmission-lines used as interconnects between two or more electronic devices act more and more as antennas, thus providing the main "access" for radiated interferences. In particular, in the automotive domain MTLs are usually unshielded, so that they play a major role in EMC emission/immunity tests. These are usually performed by measuring the intensity of the electromagnetic field at a certain distance for several orientations of the equipment under test (EUT), thus characterizing it by a sort of near-field radiation pattern. This sort of tests are usually performed in semi/fully anechoic chambers, thus simulating an ideal open area/free-space test site. Although providing reproducible results, this procedure is quite time-consuming, requiring a number of measurements for characterizing the EUT. Recently, several authors have suggested another kind of test, i.e. measuring the total radiated power (TRP) [1], [2]. The TRP has been historically measured by several means [3], all of them requiring the complete characterization of the EUT's radiation pattern which, for electrically large devices, strongly depends on the EUT geometry and excitation. A simpler and far more effective way of providing an estimation makes use of mode-stirred reverberation chambers (MSRC): a statistical evaluation of the TRP is thus provided and can be related to the maximum radiated field by considerations on the EUT electrical dimensions [4].

Although this procedure is profitable for testing an EUT, the numerical computation of the TRP still requires a great deal of time, because of the double-integration of the power density over an EUT-bounding surface. Although many analytical results are available in antenna theory for electrically short linear current-distributions (CD), only one attempt has been made (to our knowledge) for an electrically long line [5]: anyway the line was assumed to be electrically very close to the ground-plane. On the other hand, in recent years several groups have pursued new models for extending the classical TLT to a wider frequency range, in order to include higher modes, especially radiation ones [6][7][8]. Although these techniques provide a very powerful tool for high-frequency modelling of transmission-lines, their actual implementation is far from accessible to the average industrial user.

The aim of this paper is to present an analytical solution for the TRP of an overhead single-wire line, with no geometry-based approximation (e.g. an electrically short line), for the special case of a lossless complex exponential current-distribution. This case, although not valid for very "open" lines, is still valid for a wide frequency-range, allowing for a simple and effective assessment of radiation losses, and thus of the line emission/immunity features.

II. The model

The system here considered is a uniform single-wire line of finite length $\mathscr{L}$, hanging above a boundless metallic ground-plane (Fig. 1). The analytical formulation is based upon the following hypothesis:

1) the thin-wire approximation holds for the overhead conductor
2) the ground-plane is lossless and boundless
3) the medium surrounding the line is lossless
4) the current-distribution along the line can be described as the sum of two complex exponentials, as in (2), implying that the line is lossless
5) the radiation due to the discontinuities introduced by the line ends is supposed to be negligible, i.e. the line is regarded as the main source of radiation or as the main "access" for coupling to radiated interferences.

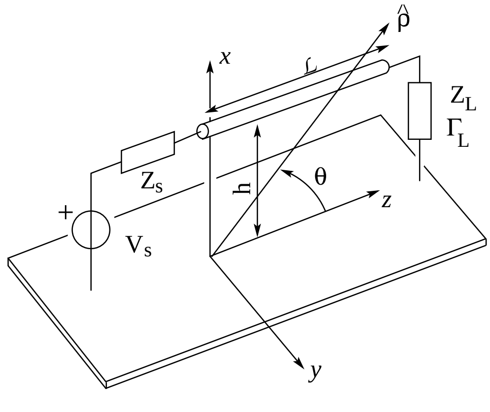

Fig. 1. The overhead line.

Even though hypothesis 4) may seem pointless, the radiation losses will be taken into account afterwards by using a two-step procedure (see section IV).

The loads and the voltage generator in Fig. 1 are just regarded as mathematical boundary conditions rather than physical connections; the actual effects of vertical risers will be discussed in section IV.

The TRP is defined by equation (1), where Ω is a closed bounding-surface embracing the entire system, $\mathbf{S}$ is Poynting's vector and $\hat{\boldsymbol{n}}$ is the unitary outward vector normal to surface Ω. Due to the hypothesis of an ideal ground-plane, this surface can be reduced to a hemisphere laying on the ground-plane ($0 < \vartheta < \pi$, $-\pi/2 < \varphi < \pi/2$), being the e.m. field identically equal to zero on and beneath the metallic bound. For the very same reason, the image principle can be invoked, so that the actual system can be substituted by an equivalent two-wire line in free-space. Furthermore, being the medium lossless, every integration surface is equivalent, yielding the same amount of TRP. Hence, an hemisphere whose radius goes to infinity can be chosen, exploiting the simpler far-field expressions for the radiated magnetic field $H(\vartheta, \varphi)$, yielding:

$$P_r = \mathrm{Re} \int_{\Omega} \mathbf{S} \cdot \hat{\boldsymbol{n}} \mathrm{d}\Omega = \int_{\Omega} \zeta_0 |H(\vartheta, \varphi)|^2 \mathrm{d}\Omega \qquad (1)$$

being ζ_0 the characteristic impedance of the surrounding medium. Variables ϑ and φ refer to figure 1: the former is the angle between the direction $\hat{\boldsymbol{\rho}}$ and the axis z, the latter is the azimuthal angle considered on the xy plane. The following lossless modal description for the CD has been used:

$$I(z) = I_0^+ (\mathrm{e}^{-\gamma z} - \Gamma_{\mathrm{L}} \mathrm{e}^{-2\gamma \mathscr{L}} \mathrm{e}^{+\gamma z}) \qquad (2)$$

where I_0^+ is the forward-travelling modal current, $\gamma = \gamma_0 \xi$ is the purely imaginary propagation constant for the wave guided by the line and Γ_{L} represents the load reflection coefficient. The parameter $\gamma_0 = \mathrm{j}k_0$ is the propagation constant of the surrounding medium, so that ξ acts as a sort of equivalent permittivity for the propagation along the line. The resulting expression for equation (1) is:

$$\begin{aligned} P_r &= \frac{|I_0^+|^2 \zeta_0}{4\pi^2} \int_0^{\pi} \int_{-\pi/2}^{+\pi/2} \frac{\sin^3 \vartheta}{|\xi^2 - \cos^2 \vartheta|^2} \cdot \\ &\cdot \sin^2(k_0 h \sin\vartheta \cos\varphi) |B(\vartheta)|^2 \mathrm{d}\varphi \mathrm{d}\vartheta \qquad (3) \\ B(\vartheta) &= (\mathrm{e}^{-\gamma_0 (\cos\vartheta + \xi)\mathscr{L}} - 1)(\cos\vartheta - \xi) + \qquad (4) \\ &- \Gamma_{\mathrm{L}} \mathrm{e}^{-2\gamma\mathscr{L}} (\mathrm{e}^{-\gamma_0(\cos\vartheta - \xi)\mathscr{L}} - 1)(\cos\vartheta + \xi) \end{aligned}$$

After some quite long algebraic manipulations, equation (4) can be rewritten as:

$$\begin{aligned} &\frac{|B(\vartheta)|^2}{2} = (\xi^2 + 1)\sigma + (\xi^2 - 1)\tau\kappa_c + \\ &\sin^2\vartheta[\tau\kappa_c - \sigma] + \\ &- \cos(k_0 \mathscr{L} \cos\vartheta)[(\xi^2 + 1)\sigma\kappa_c + (\xi^2 - 1)\tau] + \\ &\sin^2\vartheta \cos(k_0 \mathscr{L} \cos\vartheta)[\sigma\kappa_c - \tau] + \\ &-2\sigma\xi\kappa_s \cos\vartheta \sin(k_0 \mathscr{L} \cos\vartheta) \qquad (5) \end{aligned}$$

where $\sigma = 1 + |\Gamma_L|^2$, $\tau = 2\mathrm{Re}\,\{\Gamma_L \mathrm{e}^{-\gamma\mathscr{L}}\}$, $\kappa_c = \cos(k_0 \xi \mathscr{L})$ and $\kappa_s = \sin(k_0 \xi \mathscr{L})$. In order to solve equation (3), the double-integral has to be factorized into two one-dimensional integrals. An easy and straightforward solution employs McLaurin's expansion of $\sin^2 x$, which eventually yields expression (6)

$$P_r = \frac{|I_0^+|^2 \zeta_0}{4\pi^2} \sum_{n=1}^{\infty} a_n \int_0^{\pi} \frac{\sin^{2n+3}\vartheta}{|\xi^2 - \cos^2\vartheta|^2} |B(\vartheta)|^2 \mathrm{d}\vartheta \qquad (6)$$

$$a_n = -\left(\frac{k_0 h}{n}\right)^2 a_{n-1} \qquad (7)$$

being $a_0 = -\pi/2$. The terms present in integral (6) share the same structure which, though being analytically solvable, has no simple closed-form solution. Nevertheless, the special cases $\xi = 1$ and $\xi^2 - \cos^2\vartheta \cong \xi^2$ provide a closed-form result [9], which is *exact* for $\xi = 1$:

$$\begin{aligned} P_r &\cong \frac{|I_0^+|^2 \zeta_0}{\xi^4 \pi^2} \sum_{n=K+1}^{\infty} a_{n-K} \Big\{ S_{2n+1}(\tau\kappa_c - \sigma) + \\ &+ S_{2n-1}[(\xi^2+1)\sigma + (\xi^2-1)\tau\kappa_c] + \\ &- \psi_n[(\xi^2+1)\sigma\kappa_c + (\xi^2-1)\tau] + \\ &+ \psi_{n+1}\left(\sigma\kappa_c - \tau - 2\sigma\xi\frac{k_o\mathscr{L}}{2n}\kappa_s\right)\Big\} \qquad (8) \\ S_n &= \int_0^{\pi/2} \sin^n\vartheta \mathrm{d}\vartheta = S_{n-2}\frac{n-1}{n} \qquad (9) \\ \psi_n &= \frac{\mathrm{J}_{n-1/2}(k_0\mathscr{L})}{(k_0\mathscr{L}/2)^{n-1/2}} \frac{\sqrt{\pi}}{2}(n-1)! \qquad (10) \end{aligned}$$

being $K = 0$ for $\xi = 1$ and $K = 2$ otherwise. This result can be dramatically simplified by neglecting Bessel's terms ψ_n; in fact, they play a relatively minor role, as shown in the results presented in Fig. 2a.

III. Numerical validation

In order to check the validity of expression (8) as a solution for equation (1), it has been compared with the numerical integration of (1), performed by using an algorithm based on Lobatto's adaptive quadrature. In particular, this check is important for verifying whether the series expansion is fast-convergent or not, as well as for assessing the solution performance when approximating a line with $\xi > 1$. To this end, a single-wire line with $h = 10$ cm, $\mathscr{L} = 1$ m, $V_s = 1$ V and $Z_s = Z_L = 50\ \Omega$ has been considered, over a frequency range attaining 1 GHz. This means that around 1 GHz the line would be some three wavelengths long, i.e. an electrically long line.

The first check, involving the series convergence, considered the case $\xi = 1$. The results are shown in figure 2a, where the TRP obtained through numerical integration is compared with approximated solutions considering just one term and three terms with the further simplification discarding the terms related to Bessel's functions. Figure 2 proves the good convergence properties of the proposed solution, together with the fact that even the first expansion term provides a good estimation of the TRP. The fact that Bessel's function can be neglected comes from their argument $k_0\mathscr{L}$, which makes functions ψ_n negligible for an electrically long line.

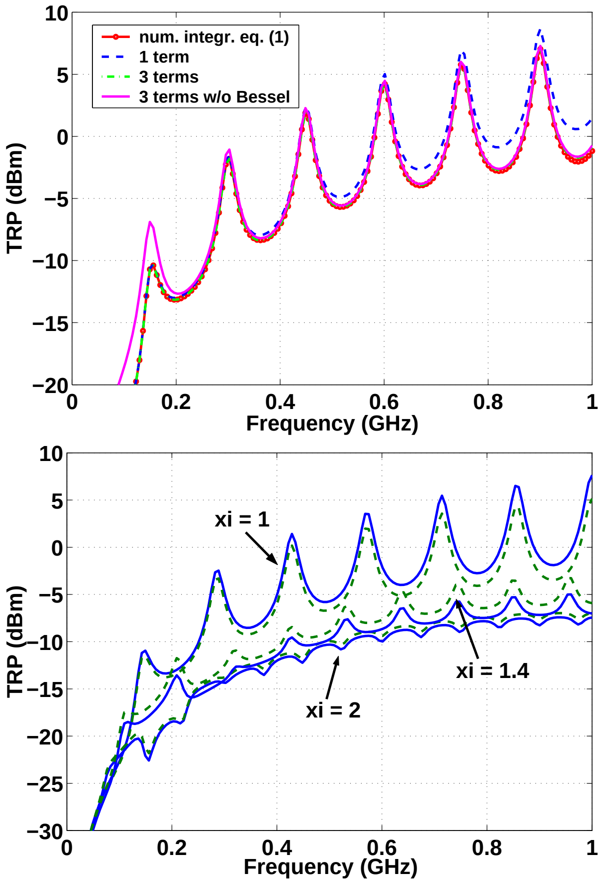

Fig. 2. The results from the convergence test for $\xi = 1$ (a) and the comparisons between the numerical integration of (1) (dashed lines) and equation (8) (solid lines).

The other check was related with the approximated solution for $\xi > 1$. In this case equation (8) provides a very good result even for $\xi = 1.4$ (figure 2b), which does not fully satisfy the condition $\xi^2 - \cos^2\vartheta \cong \xi^2$. On the other hand, whenever $\xi \cong 1$, it is possible to use equation (8) for the case $\xi = 1$, but for an actual value not equal to one. An example is shown in figure 2b, where for $\xi = 1.05$ the expansion for $\xi = 1$ has been employed: the maximum error is smaller than 3 dB. The ability of the solution to extend its agreement with the exact solution for $\xi > 1$, even in case of just a few percentage points, will be shown to be very useful for applying it to actual lines (see section V).

Another important point in favour of an analytical approach to TRP, even when flawed by strong assumptions and approximations , is the fact that the numerical evaluation of (1) is far less simple than it may appear. In fact, since we are considering an electrically long line, the radiation pattern is not "well behaved" as for an electrically short linear antenna. Indeed, it is characterized by a great number of side lobs, and thus a great number of zeros. The integration of such a function has to be carried out with due care, since even small interpolation errors around these zeros can lead to overall errors about some decibels.

IV. Application to a single-wire line: experimental validation

Equation (8) is not just a formal solution to the TRP for a lossless exponential current distribution, but it can be effectively used for assessing the TRP for an actual single-wire line. To prove that, a mock-up line as the one in figure 1 has been prepared. In this case, the line was kept hanging through two vertical risers (VRs), whereas the line's ends were connected to two N-type connectors underneath the ground-plane through two via-holes. The first connector was used for loading the line with N-type calibration loads, and the other one was connected to a coaxial cable for the line excitation. The immediate consequence of using this mock-up is the breaking of assumptions no. 2) and 5), i.e. the ground-plane has now a limited surface and VRs are bound to be an important radiation sources.

In order to measure the TRP for this system, an MSRC has been used. Although these facilities are usually employed for generating a statistically uniform and isotropic e.m. field, in this case we used it as a means for measuring the power radiated by the system along several directions without moving the receiving antenna. Indeed, adopting for the sake of simplicity a ray description, the fact that the MSRC's walls are metallic leads to multiple reflections, therefore the e.m. energy received by an antenna placed into the MSRC will include contributions from a great deal of radiation directions coming from the mock-up. Furthermore, the MSRC are equipped with a stirrer, whose task is to change the boundary conditions inside the MSRC, therefore modifying the reflection pattern and allowing the receiving antenna to measure the contributions to the radiated power as from a different set of directions. Hence, measuring the received power for a large number of stirrer's positions and computing the average received power involves an estimation of the power radiated all over an infinitely large sphere, the same as performing the flux integral expressed in (1). The experimental setup is sketched in figure 3. The stirrer

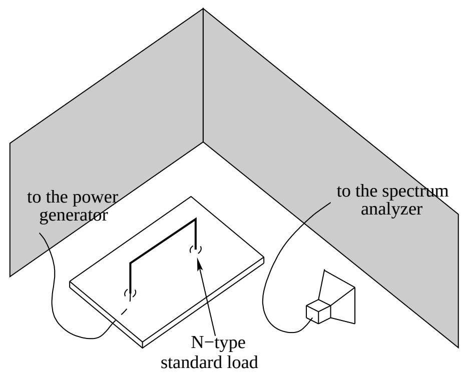

Fig. 3. A sketch of the setup employed for the experimental validation.

was set to perform a complete turn in 7 s, by 360 finite steps for each frequency investigated. Since the MSRC's dimensions were 1.9x2.65x2.9 m, the minimum frequency for which the chamber can be regarded as overmoded is about 700 MHz. The line was fed by a function generator kept outside the MSRC and connected to the line through a coaxial cable passing through a via in the MSRC's wall. In the same way a horn (SWR = 2.2 over $0.7\div 3$ GHz) was used as the receiving antenna, connected to a spectrum analyzer outside the MSRC. The entire procedure was run by a software controlling all the devices through a GPIB interface.

The result of this procedure is not yet a correct evaluation of the TRP: in fact, the MSRC insertion loss has to be assessed. The insertion loss includes cable losses, mismatches and the MSRC's own insertion loss (its Q is not infinite). Because of this, the line has been substituted by another horn antenna (identical to the receiving one), and the entire procedure has been repeated. By subtracting the resulting received power (expressed as dBm) from the first evaluation of the TRP, a good evaluation of the actual TRP is obtained.

Before applying equation (8), we need to assess the importance of VRs. To this end, the entire mock-up line, including the VRs, was simulated using NEC2 (method of moments) comparing it with the results obtained by the proposed two-step procedure (afterwards in this section) which neglect the VRs's radiation. An example of these results is shown in figure 4, for $h = 3$ cm and $\mathscr{L} = 80$ cm and the line's far-end short-circuited. As expected, due to the differential configuration of the horizontal line, its radiation efficiency is very poor at low frequencies, where the two vertical monopoles dominate the TRP. Anyway, as the electrical length of the horizontal line increases, its radiation grows stronger and the VRs can be neglected. On the other hand, as he frequency increases, the attenuation along the line gets more important, reducing the contribution coming from the horizontal line, thus leaving only the near-end VR to radiated. Although the VRs's radiation has been neglected, their effects on the propagation have been considered as proposed in [10]. Indeed, their being taken into account is very important to correctly identify the resonances position.

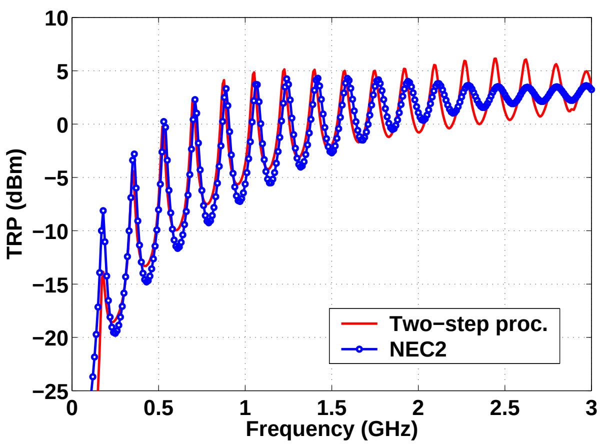

Fig. 4. The simulation results for the TRP of the mock-up line using NEC2 and the proposed model.

In order to use equation (8) with an actual line, it has be applied cunningly. Another of the basic assumptions which is no more valid is no. 4), i.e. the line can not be considered as lossless. Indeed, the very idea of a TRP means that part of the power propagating along the line is radiated away. It has been proved in [11] that the propagation along a line over an ideal ground-plane can be acceptably described by a single TEM mode as long as the ratio $h/\lambda \lesssim 1/3$. Thus, the CD can be described as the sum of two exponentials, but a more general approach requires the introduction of an exponential attenuation, as in figure 5, whereas equation (8) requires the CD to be uniform. Nevertheless we are not interested in the actual CD, but rather in the effect of the CD for the sake of TRP. So we could consider an equivalent CD, with no loss over a certain length, exactly in the same way as for the current excited by an e.m. field penetrating into a conductive medium; indeed, we can regard this current as uniformly distributed on an equivalent portion known as skin-effect penetration depth. The same idea has been applied in figure 5 to the CD. The problem with

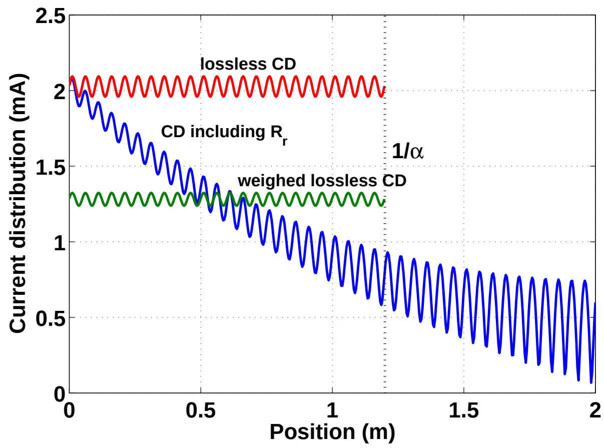

Fig. 5. An example of attenuated CD and the equivalent CDs employed by the model.

this approach is that the attenuation constant has to be assessed. Being related to the radiated field, the radiation losses are dependent on the entire CD. This means that it does not suffice just to know the line's cross-section as for the p.u.l. parameters: therefore they cannot be generally treated as ohmic losses. Furthermore, the presence of discontinuities such as VRs could introduce privileged sources of radiation, thus making pointless the definition of a p.u.l. radiation resistance. Indeed, that would mean to spread all over the entire line an effect that is perhaps concentrated in a limited region, such as for a VR.

Anyway, for the special case of an infinitely long uniform line, the radiation losses act as ohmic losses [7], therefore a p.u.l. radiation resistance can be defined and included in the p.u.l. impedance, thus accounting for the radiation losses. The same procedure can be pursued for a finite-length line [12], as long as the VRs's contribution is negligible (we are neglecting all type of fringing effects): in this case the line finiteness is taken into account by the CD itself, which includes a backward-travelling wave. Hence, what is needed is to compute the radiation resistance R_r. This can be accomplished by using the following definition:

$$R_r = \frac{P_r}{|I_0^+|^2} \frac{1}{(1 + |\Gamma_{\mathrm{L}}|)^2} \tag{11}$$

referring to the CD introduced in (2), i.e. with no losses. In fact, even though this CD does not necessary suit the actual CD, it provides an acceptable estimation of R_r. Now it is possible to compute the attenuation constant as:

$$\alpha = \mathrm{Im}\left\{\sqrt{(z_e + R_r/\mathscr{L})y_e}\right\} \tag{12}$$

where z_e and y_e are respectively the p.u.l. impedance and admittance for the TEM mode (or quasi-TEM considering ohmic losses). Equation (8) can be used by considering the modal parameters $I_0^{\pm}$ computed with the new CD including radiation losses, but for a reduced length $\mathscr{L}_{eq} = 1/\alpha$ (obviously $\mathscr{L}$ acting an an upper-bound), as in figure 5. Moreover, the resulting P_r is weighed down by a factor w^2:

$$w = \frac{1 - \mathrm{e}^{-\alpha \mathscr{L}_{eq}}}{\alpha \mathscr{L}_{eq}} \tag{13}$$

accounting for the attenuation along the equivalent length $\mathscr{L}_{eq}$, thus considering the equivalent weighed CD as in figure 5. Some experimental results are shown in figure 6:

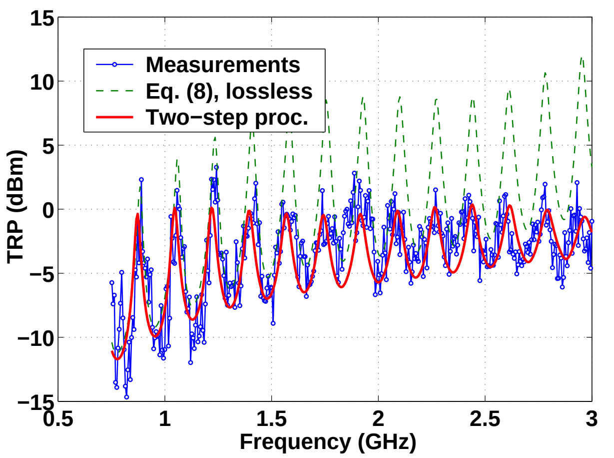

Fig. 6. Some results from the experimental validation of the proposed two-step procedure. The fluctuations in the low-frequency range are due to the limited number of cavity modes for the MSRC.

the line under test was short-circuited at the far-end with $h = 3$ cm, $\mathscr{L} = 80$ cm; the wire's diameter was 1.5 mm and the ground-plane dimensions were 60 cm and 120 cm. The TRP was studied in the frequency-range 700 MHz ÷ 3 GHz, hence considering the line as electrically long. The agreement between the proposed model and the experimental result is quite good, with an average absolute error about 2 dB. Figure 6 also points out the importance of assessing the radiation resistance before computing the actual TRP.

The procedure here described, although apparently lengthy, is actually very simple and straightforward, being basically made up by two steps, i.e. the evaluation of the line radiation resistance and the actual TRP, through the definition of an equivalent CD. Because of this simplicity and straightforwardness, we think that this procedure is suitable for industrial purposes, as a compromise between a theoretically sound description and a far more approximated solution. Its main limitation is its being based on a quasi-TEM propagation model; hence, it is valid as long as the ratio h/λ holds [11].

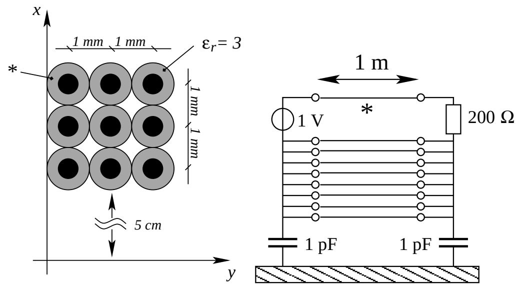

Fig. 7. The 9-wire test case for the application to an MTL. No VRs are present.

V. An Application to MTLs

Since a single-wire line is not very common in industrial applications, we had to focus our attention on MTLs. Although equation (8) holds just for a single-wire line, it can be usefully employed for an MTL above a metallic ground-plane. Actually, in EMC the most dramatic effects regarding the radiated emission/immunity problems are due to the common-mode (CM) current, or bulk current $I_b(z)$. Let us consider the modal description for the CD along a N-wire MTL:

$$\mathbf{I}(z) = \mathbf{T}\left[\mathbf{P}^{+}(z)\mathbf{I}_{m0}^{+} - \mathbf{P}^{-}(z)\mathbf{I}_{m0}^{-}\right] \tag{14}$$

$$\begin{aligned} I_b(z) &= \sum_{i=1}^{N} I_i(z) = \left(\sum_{i=1}^{N}\mathbf{T}_i\right)\mathbf{I}_m(z) \\ &= \mathbf{Q}\mathbf{I}_m(z) = \sum_{i=1}^{N}\sum_{k=1}^{N} T_{ik} I_{m,k}(z) \end{aligned} \tag{15}$$

where $\mathbf{T}$ is a square matrix relating the modal currents to the physical ones, $\mathbf{P}^{\pm}(z) = \mathrm{diag}(\exp(\mp\boldsymbol{\gamma}z))$ are the propagation matrices, $\boldsymbol{\gamma}$ is the vector of the line's propagation constants and $\mathbf{I}_{m0}^{\pm}$ are the excitation factors for the forward- and backward-travelling modal-currents. The e.m. field radiated by an MTL is mainly due to the CM current, as long as the line's conductors are "packed" together, being the cross-section position of the i-th wire identified as $y = d_i$ and $x = h_i$. This assumption requires $k_0|\bar{h} - h_i| \ll 1$ and $k_0|\bar{d} - d_i| \ll 1$ $\forall i \in [1, N]$, where $\bar{h}$ and $\bar{d}$ stand for the averages on h_i and d_i.

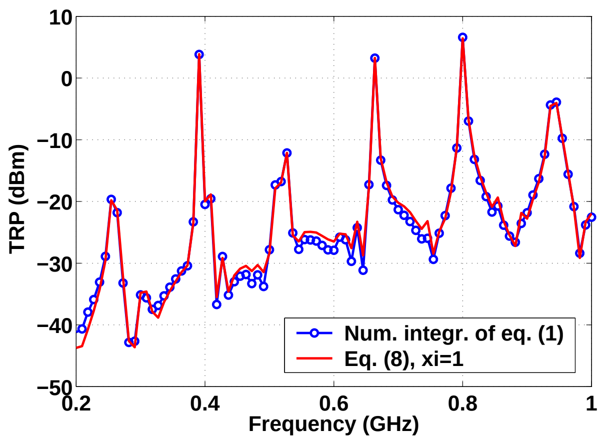

Fig. 8. The numerical validation for the 9-wire test case.

Let us consider an overhead uniform MTL: the magnetic field can be expressed, in the far-field, as:

$$\begin{aligned}
\mathbf{H}(\vartheta,\varphi) &= \hat{\boldsymbol{\varphi}}\frac{\gamma_0}{4\pi\rho}\sin\vartheta \mathrm{e}^{-\gamma_0\rho}\sum_{i=1}^{N}\mathrm{e}^{-\gamma_0 h_i \hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{\rho}}}\mathrm{e}^{-\gamma_0 d_i \hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{\rho}}}\,\cdot \\
&\cdot \int_\Gamma \sum_{k=1}^{N} T_{ik} I_{m,k}(z)\mathrm{e}^{-\gamma_0 z\hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{\rho}}}\mathrm{d}\Gamma \qquad (16) \\
&\cong \hat{\boldsymbol{\varphi}}\frac{\gamma_0}{4\pi\rho}\sin\vartheta \mathrm{e}^{-\gamma_0\rho}\mathrm{e}^{-\gamma_0\overline{h}\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{\rho}}}\mathrm{e}^{-\gamma_0\overline{d}\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{\rho}}}\,\cdot \\
&\cdot \int_\Gamma I_b(z)\mathrm{e}^{-\gamma_0 z\hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{\rho}}}\mathrm{d}\Gamma \qquad (17)
\end{aligned}$$

The integration-path Γ describes the entire CD and its image. The right-hand approximation holds for a "packed" line using (15), and shows that the main contribution to the radiated field is due to the CM current. The importance of this result lays in the fact that whenever the CM current can be approximated as the CD along a single-wire line, equation (8) can be usefully employed for estimating the TRP for an MTL. Indeed, this usually holds in several cases of actual interest, e.g. for unshielded MTLs employed in the automotive domain. For these cases the single-wire CD of the CM can be derived from the modal description:

$$I_b(z) = I_{b0}^{+}\mathrm{e}^{-\gamma_b z} - I_{b0}^{-}\mathrm{e}^{+\gamma_b z} \qquad (18)$$

$$I_{b0}^{\pm} = I_{m0,j}^{\pm}\sum_{i=1}^{N} T_{i,j} \qquad (19)$$

where γ_b corresponds to the CM's propagation constant in $\boldsymbol{\gamma}$, as its j-th element. Furthermore, for a line as the one in Fig. 7, the CM is always identified as the one with the smallest propagation constant. This is due to the fact that the electric field force-lines are mostly in air rather than in the wires' coating, whereas the differential modes present the opposite situation.

On the other hand, for a homogeneous medium $\gamma_i = \gamma\ \forall i \in [1, N]$, so that the previous procedure is no more available. In this case $\mathbf{P}^{\pm}(z) = \exp(\mp\gamma z)\mathbb{I}$ so that:

$$I_b(z) = \mathbf{Q}[\mathbf{I}_{m0}^{+}\mathrm{e}^{-\gamma z} - \mathbf{I}_{m0}^{-}\mathrm{e}^{+\gamma z}] = I_{b0}^{+}\mathrm{e}^{-\gamma z} - I_{b0}^{-}\mathrm{e}^{+\gamma z} \qquad (20)$$

This procedure has been applied to the line shown in Fig. 7, characterized by a dielectric coating with $\epsilon_r = 3$, computing matrices $\mathbf{T}$ and $\boldsymbol{\gamma}$ by a moment method [13]. The analytical solution is compared with the numerical computation of the TRP, by inserting equation (16) into (1). The results of the validation are shown in Fig. 8, based on equation (8) and (19) for 3 expansion terms. For this experimental validation we have just considered a lossless line, being the aim just to verify the possibility of approximating the MTL's TRP through a single-wire equivalent line.

The agreement is quite good, with a maximum error about 1.5 dB with respect to the approximation $\xi \cong 1$, actually being $\xi \cong 1.02$ for the CM; for the other modes $\xi_i \in [1.39, 1.58]$. The procedure here proposed could also be used for estimating the line's immunity with respect to an external interference, thanks to the reciprocity theorem [2], assessing the CM current induced along the line. This

VI. Conclusions and further developments

We have presented an analytical solution for the TRP for a single-wire line above an ideal ground-plane. The solution, expressed as a series expansion, appears to be fast-convergent, requiring just 3 terms for $h/\lambda < 1/3$ ($h = 10$ cm at 1 GHz). In particular, the solution does not require the line to be electrically short. A two-step procedure has also been envisaged for applying the proposed model to an actual single-wire line, whose validity has been verified through experimental investigation.

Moreover, it has been shown that the proposed formulation can be effectively used for the estimation of the radiated emission/immunity of a unshielded MTL, which is an important feature when dealing with EMC issues in the automotive domain. The extension of the model to non-packed or open MTL is currently in progress, i.e. for a class of lines were the DM's contribution becomes important. Besides, the same procedure we are using for dealing with open MTLs might be used for including the contribution of VRs, at least on a speculative level; unfortunately, in this case a more complex multi-step approach should be envisaged, since the line could be no more regarded as uniform.

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